ABSTRACT
A generalized expression defining the input admittance of any single op. amp. active-RC network is formulated. It is used to synthesize an ideal negative inductor using only one capacitor and two resistors. Due to practical limitations, the quality factor of the inductor is finite, but high. Also, an extra resistor has to be added to the network. However, the Q-factor can be made infinite at any one specific frequency.

INTRODUCTION
Active-RC simulations of inductors have been developed for use in building ladder structure filters with low sensitivities, derived from the filters' passive LC realization [1], [2]. These simulations either employ two op. amps. [3] or a single op. amp. [4], [5], [6]. The single op. amp. realizations usually require two cancellation conditions to eliminate some coefficients in the input impedance expression; thus reducing their attractiveness due to the need for precision components and exact tracking with temperature and aging. Recently, a new single op. amp. realization of an RLC branch was proposed [7], requiring only one cancellation condition.

The negative inductor simulation discussed in this paper does not require any cancellation condition. Besides a single op. amp., it uses minimum number of components, and hence is a canonical realization. Together with negative resistance and negative capacitance realizations [8], the negative inductor completes a class of negative impedance elements and presents a new challenge to circuit designers in proposing practical applications involving these devices.

DERIVATION OF THE INPUT ADMITTANCE
Any single op. amp. active RC network can be represented in the form of the generalized structure shown in Fig. 1. Assuming that ports 3 and 4 are at open circuit conditions, the dependent voltage source at port 2 is obtained from (1).

\[ V_2 = V_1 \frac{T_{41} - T_{31}}{A(s)} + \frac{T_{32} - T_{42}}{A(s)} \]  

where 

\[ A(s) = \frac{A_{0}}{1 + A_{0} \frac{s}{\omega_T}} \]  

is the open loop voltage gain of the op. amp. and \( \omega_T \) is the gain-bandwidth product.

For the RC network which is then excited by two voltage sources,

\[ T_{41} = \frac{V_4}{V_1} \quad T_{42} = \frac{V_4}{V_2} \quad T_{31} = \frac{V_3}{V_1} \quad T_{32} = \frac{V_3}{V_2} \]

Substituting for \( V_2 \) from (1),

\[ Y_{in} = \frac{I_1}{V_1} = y_{11} + y_{12} \]  

The above expression then defines the input admittance of any single op. amp. active-RC network. It can be used not only in the analysis of active filters and active impedances, but can also provide further insight during the process of synthesizing any required active impedance specifications.

SYNTHESIS OF A NEGATIVE INDUCTOR
The RC network shown in Fig. 2-a has the following characteristics:

\[ y_{11} = \frac{1}{R_2} \]  

\[ \frac{y_{12}}{y_{11}} = 1 \]  

\[ T_{42} = T_{31} = 0 \]  

\[ T_{41} = 1 \]

and

\[ T_{32} = \frac{R_1}{1 + R_1 Cs} \]

Assuming that the op. amp. is ideal, with \( A(s) \) tending to \( \infty \), equation (3) then gives:

\[ Z_{in} = -R_2 \frac{R_1}{1 + R_1 Cs} \]

Thus an ideal negative inductor can be actively realized with the minimum number of components, namely one capacitor and two resistors. The complete structure is shown in Fig. 2-b. No cancellation conditions are required which make the need for precision elements unnecessary.
PRACTICAL CONSIDERATIONS

The active realization of an ideal negative inductor, shown in Fig. 2-b, suffers from the drawback that it is essentially operating under open loop conditions with respect to d.c. voltages. Thus any slight d.c. offset due to temperature variations or unbalanced inputs can result in amplifier saturation. This difficulty can be overcome by allowing for a very small amount of d.c. negative feedback through resistor $R_3$, as shown in Fig. 3. It is necessary here to add the condition

$$ R_3 \gg R_1 $$

As will be observed later, $R_3$ can be useful in controlling the quality factor of the inductor. In fact $Q$ can be made infinite at one specific finite frequency.

Taking into account a practical single-pole model of the op. amp., equation (2), the modified realization of the negative inductor in Fig. 3 can be analyzed as outlined above. Here

$$ T_{32} = \frac{\alpha + \frac{s}{s_o}}{1 + \frac{s}{s_o}} $$

where

$$ \alpha = \frac{R_1}{R_1 + R_3} $$

and

$$ \omega_o = \frac{R_1 + R_3}{R_1 R_3 C} $$

The input impedance from equation (3) is

$$ Z_{in} = L_{eq} \cdot R_{eq} $$

where

$$ L_{eq} = -\frac{R_2}{\omega_o} \left[ 1 + \alpha + \frac{2}{\omega_o} \cdot \omega \left( 1 - \frac{\omega^2}{\omega_0^2} \right) \right] $$

and

$$ R_{eq} = -\frac{R_2}{\omega_o} \left[ \alpha - \frac{1}{\omega_o^2} - 2 \frac{\omega^2}{\omega_o^2} \cdot \left( \frac{1}{2\omega_o} + \frac{\omega}{\omega_0} \right) \right] $$

The above relations are first-order approximations under the conditions

$$ \omega_o \ll \omega_1 $$

and

$$ \omega \ll \omega_1 $$

(12a)

(12b)

It is important to note that the frequency of operation, $\omega_1$, should preferably be less than $\omega_0$ or at most of the same order of magnitude as $\omega_0$.

Neglecting the terms that contain $1/\alpha$ with respect to other terms in equations (10) and (11), the quality factor of the negative inductor can be defined as:

$$ Q = \frac{\omega}{s_o} \cdot \frac{1 + \alpha + \frac{\omega}{\omega_0} \left( 1 - \frac{\omega^2}{\omega_0^2} \right)}{\alpha - 2 \frac{\omega^2}{\omega_0^2} \frac{\omega}{\omega_1}} $$

(13)

$$ = \frac{Q}{R_3} = \frac{1}{1 - \frac{\omega^2}{\omega_o^2}} $$

(15a)

$$ = Q \frac{M}{R_3} = \frac{1}{1 - \frac{\omega^2}{\omega_o^2}} $$

(15b)

where

$$ Q = \frac{\omega}{s_o} \left[ 1 + \frac{\omega}{\omega_1} \left( 1 - \frac{\omega^2}{\omega_0^2} \right) \right] $$

(14)

and

$$ \omega = \frac{\alpha}{2} - \omega \cdot \omega_1 $$

(15)

The graph in Fig. 4 shows the variation of $Q$ with frequency. The two terms of equation (13-a) are plotted separately. The multiplication factor $M$ is always higher than unity for

$$ 0.707 \omega_o \ll \omega \ll \omega_1 $$

being infinity at $\omega = \omega_o$.

For those frequencies below 0.707 $\omega_o$, where the magnitude of $M$ is less than unity, the equivalent resistance $R_{eq}$ is negative and can thus be compensated externally. Hence resistor $R_3$ can be used beneficially to improve the quality factor of the negative inductor.

Table 1 summarizes the sensitivity parameters of the negative inductor. $L_{eq}$ have very low sensitivities to passive and active elements variations. On the other hand, the series resistor $R_{eq}$ and correspondingly the quality factor of the inductor are extremely sensitive to passive and active elements variations at the frequency $\omega_0$ (i.e. at $Q = 1$). However, as can be observed from Table 1, these sensitivities can become relatively small when

$$ \omega_o \ll \omega \ll \omega_1 $$

DESIGN EXAMPLES

The design parameters $\omega_0$ and $\omega_1$ (or $\alpha$) can be chosen arbitrarily depending on the required quality factor and the frequency range of operation of the negative inductor to be realized. Two examples will be discussed showing the different choice possibilities.
CONCLUSIONS

1. The required negative inductor has an inductance
   \[ L = 0.1 \text{ H} \]

   It should have a quality factor better than 500 for frequencies in the range
   \[ 500 \text{ Hz} < f < 1 \text{ kHz} \]

   A uA741 op. amp. \((f_p = 1 \text{ MHz})\) and a 10nF capacitor are used.\(^{1}\) Taking \(f_p\) to be \(500 \text{ Hz} \sqrt{2}\)
   and \(f_o\) to be 1kHz, equation (15) gives:
   \[ \alpha = 0.25 \times 10^{-4} \]
   Then from (7) and (8),
   \[ R_1 = 15.92 \Omega \]
   \[ R_3 = 63.66 \Omega \]

   Finally, equation (10) gives:
   \[ R_2 = 628 \Omega \]

   For this inductor
   \[ Q | f = 500 \text{ Hz} | = 2000 \]
   \[ Q | f = 1 \text{ kHz} | = -571 \]

2. To design a similar inductor with a quality factor better than 250 in the frequency range
   \[ 250 \text{ Hz} < f < 500 \text{ Hz} \]

   and using the same op. amp. and capacitor, \(f_o\) is taken to be 1.59 kHz and \(f_p\) is taken to be 707 Hz, then:
   \[ \alpha = 0.63 \times 10^{-3} \]
   \[ R_1 = 10 \Omega \]
   \[ R_3 = 15.92 \Omega \]

   and \( R_2 = 1k\Omega \)

   Here
   \[ Q | f = 250 \text{ Hz} | = 286 \]
   \[ Q | f = 500 \text{ Hz} | = 1000 \]

   In this example it is important to note that the equivalent resistance is negative.

   The second design example above was built and tested, using 15 components. Its actual performance agreed extremely well with that expected.

REFERENCES


ACKNOWLEDGMENT

The author wishes to thank M.T. Ghorab, J. Siluberg, and G. Barsony for valuable discussions and comments.
### TABLE 1

**SENSITIVITY PARAMETERS**

<table>
<thead>
<tr>
<th>$X$</th>
<th>$\frac{\mu_{eq}}{S_{eq}}$</th>
<th>$\frac{L_{eq}}{S_{eq}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_2$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$R_1$</td>
<td>$1 - \alpha$</td>
<td>$1 + \alpha \left(1 - \frac{\omega_0^2}{\omega_p^2}\right)$</td>
</tr>
<tr>
<td>$R_3$</td>
<td>$\alpha + \frac{1}{\omega_p^2 - 1}$</td>
<td>$\alpha^2 \left(2 - \frac{\omega_0^2}{\omega_p^2}\right)$</td>
</tr>
<tr>
<td>$C$</td>
<td>$1 + \frac{1}{\omega_p^2 - 1}$</td>
<td>$1 - \frac{\omega_0}{\omega_p}$</td>
</tr>
<tr>
<td>$A_0$</td>
<td>$\frac{1}{A_0} \cdot \frac{1}{\alpha \left(\frac{\omega_p^2}{\omega_0^2} - 1\right)}$</td>
<td>$-\frac{2}{A_0}$</td>
</tr>
<tr>
<td>$\omega_2$</td>
<td>$-1 - \frac{1}{\omega_p^2 - 1}$</td>
<td>$-\frac{\omega_0}{\omega_2} \left(1 - \frac{\omega_0^2}{\omega_p^2}\right)$</td>
</tr>
</tbody>
</table>

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**FIG. 1**

A generalized RC-active network containing one op. amp.

**FIG. 2-a**

The RC network required for simulating a negative inductor.

**FIG. 2-b**

The ideal negative inductor, RC-active simulation.
Variation of the quality factor for the ideal and practical negative inductor with frequency.