

## Characterizing the Capacity of Wireless Ad Hoc Networks

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**Abstract**—This paper discusses an overview of metrics for capacity evaluation of Ad hoc wireless networks. The capacity metrics is based on statistical approach incorporate aspect from the physical layer and from the network layer. These metrics are suitable for network design and parameter optimization. Whereas the capacity metrics based on network scalability describe how network capacity behaves when the number of nodes in the network grows. Therefore even though the scaling laws are rather pessimistic, it can be used to design of more appropriate transmission schemes.

**Keywords**- component; Expected forward progress, Information efficiency, Transmission capacity, Aggregate multi-hop information efficiency, Transport capacity, the protocol interference model, the physical interference model etc.

### I. INTRODUCTION

For the modeling and measuring the capacity of ad hoc networks [1], a large number of metrics have been proposed for characterizing the capacity of ad hoc networks under different conditions and emphasizing different aspects of the network. The investigation of the relationship between capacity and transmission radius in a network of packet radios operating under ALOHA protocol are based on the metric called expected forward progress, defined such way to capture the trade off relating the one-hop throughput and the average one-hop length [2]. Decreasing one-hop length may increase throughput due to link quality improvement and also may decrease throughput due to a larger traffic and a higher contention level caused by consequent larger number of hops between source and destination [3]. The concept of information efficiency defined as the product of the expected forward progress and the spectral efficiency of the transmission systems takes into account of channel reuse and multi-hop transmissions leading to new metric, named aggregate multi-hop information efficiency [4]. Based on a similar concept of information efficiency, the metric transmission capacity is related to the optimum density of concurrent transmissions. Transmission capacity is the area spectral efficiency of successful transmissions from the optimal contention density [5]. The transmission capacity metrics have in common their statistical nature of several mechanisms related to wireless communications, such as the interaction among nodes sharing a given channel and the propagation effects.

Based on a deterministic approach to characterize the capacity of ad hoc networks and on the behavior of capacity scaling laws, the concept of transport capacity relates transmission rate and source-destination distance [6]. The formulation of transport capacity from the

perspective of the requirements for successful transmission is described according to two interference models: the protocol interference model, which is geometric based, and the physical interference model, based on signal-to-interference ratio requirements [7]. When the number of nodes grows, the behavior of network capacity show that the per-node throughput decreases as  $O(1/\sqrt{n})$ , when  $n$  is the number of nodes in the network [8]. This chapter provides an overview of capacity metrics for wireless ad hoc networks.

### II. STATISTICAL BASED CAPACITY METRICS

The inherent random nature of ad hoc networks suggests a statistical approach to quantify capacity of such networks. A Statistical approach is very useful for the design of practical communication systems, when a set of quality requirements is imposed by the user application. Some statistical-based capacity metrics, namely expected forward progress, information efficiency, transmission capacity and aggregate multi-hop information efficiency metrics are discussed in this chapter.

#### A. Expected Forward Progress

Expected forward progress (EFP) is measured in meters and is defined as the product of the distance travelled by a packet toward its destination and the probability that such packet is successfully received.

$$EEP = d \times (1 - P_{out}) \quad (1)$$

Where  $d$  is the transmitter-receiver separation distance and  $P_{out}$  is the outage probability i.e., the probability that the bit error rate is higher than a given threshold.

#### B. Information Efficiency

Information efficiency is defined as the product of Expected forward progress (EFP) and spectral efficiency  $\eta$  of the link connecting transmitter and receiver nodes,

$$IE = \eta \times d \times (1 - P_{out}) \quad (2)$$

IE quantifies how efficiently the information bits can travel towards its destination. To capture the trade off by the information efficiency, a transmission system in which modulation and error correcting coding techniques will be selected to optimize the IE of the network. If a modulation technique with large cardinality is used, then the spectral efficiency of the system increases, at a higher minimum required signal-to-interference plus noise ratio (SINR) to achieve a given packet error probability. This higher required SINR increases the outage probability  $P_{out}$ . Error

correcting coding techniques reduces minimum required SINR at the higher bandwidth, reducing therefore the spectral efficiency of the transmissions. These trade off are captured by the information efficiency metric, allowing for a joint system design involving modulation, coding, transmission range, among other parameters [10].

### C. Transmission Capacity

Transmission capacity (TmC) metric single-hop ad hoc networks is defined as the product of the density of successful links and their communication rates, subject to a constraint on the outage probability [11].

$$TmC = \eta \times \lambda \times (1 - P_{out}) \quad (3)$$

Where,  $\lambda$  is the density of active links in the network. TmC quantifies the spatial spectral efficiency of the network, capturing in its formulation the effects of active links density on the outage probability. With a high density of concurrent transmissions, information flow in the network is also higher indicated by a high TmC. However the downside of a high density of active links is an increase in the interference level, leading to a higher outage probability and a lower transmission capacity.

### D. Aggregate Multi-hop Information Efficiency

Aggregate Information Efficiency (AIE) is defined as the sum of Information Efficiency (IE) of active links in the network per unit area [9]. Aggregate Multi-hop Information Efficiency (AMIE) is to abstract multi-hop links and evaluate AMIE based on end-to-end performance of multi-hop links.

$$AMIE = d \times \eta \times \lambda \times (1 - P_{out}) h \quad (4)$$

Where  $h$  is the average number of hops between source and destination. The main advantage of AMIE is to be more flexible and general than other similar metrics.

## III. CAPACITY SCALING LAWS

Capacity scaling laws of wireless networks is defined how capacity scales as the number of nodes in the network grows [12]. This is an important prospect to investigate how several intrinsic aspects of wireless communication such as interference, channel reuse and resource limitation affects the performance of the network. Throughput, measured in bits per second, is a metric of capacity of communication networks. In ad hoc networks, source and destination nodes may be far apart, such that direct communication (single hop) is not possible, requiring a multi-hop connection, with neighboring nodes acting as relays. Multi-hop connection leads to a traffic increase, as a given packet is transmitted several times before reaching its final destination. Therefore, source-destination separation distance must be taken into account when characterizing capacity in wireless ad hoc networks. To do so, Transport Capacity, measured in bit-meter per second is used. By considering a network with transport capacity of  $T$  bit-meter per second, the rate between two nodes spaced one meter away from each other is  $T$  b/s. If the distance between the nodes is doubled, the rate decreases to  $T/2$  b/s.

### A. Transport Capacity under the Protocol Interference Model

A network of  $n$  immobile nodes, which can act simultaneously as source, relay or destination is considered. These  $n$  nodes are arbitrarily located in a planar disk of unity area. The positions of the nodes can be adjusted to satisfy the conditions for successful transmissions imposed by interference model [13]. Every node selects randomly another node as the destination of its bits.

Transport capacity  $TA$  of an arbitrary network with  $n$  nodes under the protocol model,

$$TA = \Theta(W \sqrt{n}) \text{ bit. Meter/s} \quad (5)$$

This means that the transport capacity per node is  $\Theta(W \sqrt{1/n})$  bit. Meter/sand goes to zero as the number of nodes increases.

Under the protocol reference model, disks of radius equals to  $\frac{\Delta}{2} |X_i - X_{R(i)}|/2$  centred at receiver nodes of successful links are disjoint as shown in Fig.(1(a)) and Fig.(1(b)).

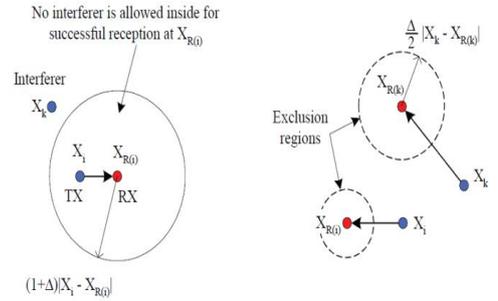


Figure 1 The protocol model:(a) Disk around receiver  $X_{R(i)}$  must be free of interfering nodes for correct reception at node  $X_{R(i)}$ ; (b) Two links are successful if the corresponding exclusion regions are disjoint.

Each successful link consumes a fraction of network area and the sum of area of disks of all successful links is upper limited by the network area. Neglecting the border effects (i.e. when nodes are close to the boundary of network area) as shown in Fig. (2).

$$\sum_{i \in T(t)} \pi \left( \frac{\Delta}{2} d_i \right)^2 \leq 1 \rightarrow \sum_{i \in T(t)} d_i \leq \frac{4}{\pi \Delta^2} \quad (6)$$

Where  $d_i$  is the T-R separation distance

$|X_i - X_{R(i)}|$  of the  $i$ -th T-R pair, and  $T(t)$  is the set of successful links at time  $t$ .

By assuming that all sources transmitting at rate  $W$ , the transport capacity  $TA$  of the network at a given time  $t$  in upper bound,

$$T_A = W \sum_{i \in T(t)} d_i \leq \sqrt{\frac{2W}{\pi \Delta}} \sqrt{n} \quad (7)$$

Or

$$T_A = O(W \sqrt{n}) \text{ bit meter/s} \quad (8)$$

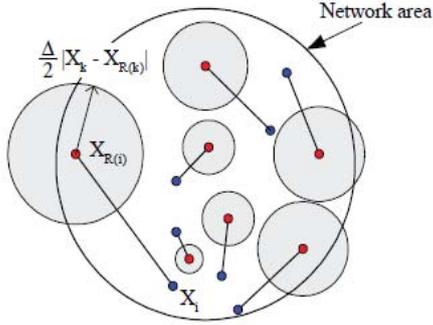


Figure 4. Arbitrary network under the protocol interference model: successful links correspond to disjoint disks.

### B. Transport Capacity under the Physical Interference Model

The transport capacity under the physical interference model,

$$T_A = O\left(Wn^{\frac{\alpha-1}{\alpha}}\right) \text{ bit } \frac{\text{meter}}{\text{s}} \quad (9)$$

$$T_A = W \sum_{i \in T(t)} d_i \leq \frac{W}{\sqrt{\pi}} \left(\frac{2\beta+2}{\beta}\right) 1/\alpha n^{\frac{\alpha-1}{\alpha}} \quad (10)$$

If the capacity is equitably shared among all sources, the transport capacity per node is,

$$T_A = O\left(\frac{W}{n^{1/\alpha}}\right) \quad (11)$$

And it goes to zero as  $n$  increases. This upper bound indicates that a larger path loss exponent  $\alpha$  leads to a higher capacity. Therefore a larger  $\alpha$  means stronger signal attenuation and reduced interference.

### C. Throughput Capacity under the Protocol Interference Model

Throughput capacity in bits per second of a random network under the protocol interference model in upper bound,

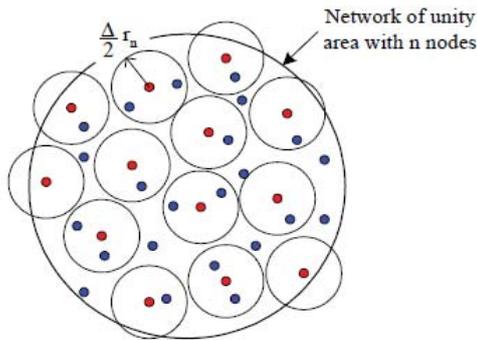


Figure 6. The protocol model: Disks around active receivers must be disjoint.

$$\lambda(n) \leq \frac{cW}{\sqrt{n \log n}} \quad (12)$$

Now, a network with  $n$  nodes randomly placed on a disk of unity area and all nodes transmitting with a common transmission range  $\Delta r_n$  is considered. In order to make sure that no node is isolated in the network,  $\Delta r_n$  must be larger than  $\sqrt{\log n / \pi n}$ . Under the protocol interference model, successful transmissions require that disks of radius  $\Delta r_n / 2$ , centered at receivers, must be disjoint, as shown in Fig.(3).

Therefore, the number of successful transmissions  $N_s$  within a disk of unity area in upper bound,

$$N_s < \frac{4}{\pi \Delta^2 r_n^2} \quad (13)$$

Therefore, the aggregate number of bits transmitted per second in the network cannot be larger than

$$\frac{4W}{\pi \Delta^2 r_n^2} \quad (14)$$

Where  $W$  is the common transmission rate of the individual transmissions.

It is considered that source nodes choose their destination nodes randomly and denote  $\bar{L}$  the average source-destination separation distance.  $\bar{L}$  does not depend on the number of nodes in the network. Therefore, the average number of hops between source and destination in lower bound as shown in Fig. (4).

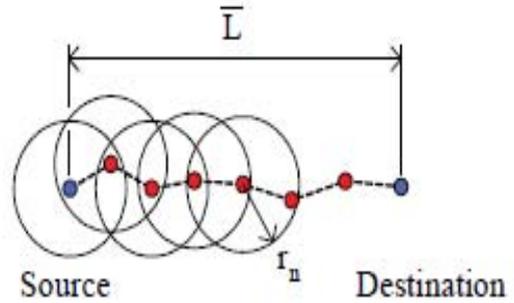


Figure 5. The protocol model: average number of hops between source and destination

If each source generates bits at rate  $\lambda(n)$ , then the average number of bits transmitted by  $n \lambda(n) \bar{L} / r_n$  and must satisfy

$$n \lambda(n) \frac{\bar{L}}{r_n} \leq \frac{4W}{\pi \Delta^2 r_n^2} \quad (15)$$

Finally, under the protocol interference model, throughput capacity per bits per second,

$$\lambda(n) \leq \frac{cW}{(1+\Delta)^2 \sqrt{n \log n}} \quad (16)$$

The order of throughput capacity of random networks under the protocol interference model,

$$\lambda(n) = \Theta(W/\sqrt{n \log n}) \quad (17)$$

#### D. Throughput Capacity under the Physical Interference Model

An upper bound on the throughput for random network under the physical interference model can be derived using the upper bound on the throughput for the case under the protocol interference model. Successful links ( $X_i, X_{R(i)}$ ) in a random network under the physical interference model are also useful under the protocol interference model, for appropriate values of  $\Delta$  and  $\beta$ . Therefore, an upper bound on the throughput for the protocol model also holds for the physical model. Therefore, for a random network under the physical interference model the throughput in upper bound,

$$\lambda(n) < \frac{cW}{\sqrt{n}} \quad (18)$$

#### IV. CONCLUSION

To characterize the capacity of wireless ad hoc networks, the capacity metrics can be classified into two groups: metrics based on a statistical approach, and metrics based on network scalability. In the first group, capacity metrics incorporate aspect from the physical layer (e.g. modulation parameters, spectral efficiency etc.) and from the network layer (e.g. spatial reuse, number of hops etc.). Therefore, these metrics are suitable for network design and parameter optimization. Apart from it, some capacity metrics like Expected Forward Progress, Information Efficiency, Transmission Capacity and Aggregate Multi-hop Information Efficiency. In the second group, the capacity metrics describe how network capacity behaves when the number of nodes in the network grows. The scaling laws are related to transport capacity and throughput capacity in terms of the protocol interference model and the physical interference model respectively. Therefore, even though the resulting scaling laws are rather pessimistic (per-node capacity vanishes as the size of the network increases), the results can be used for the design of more appropriate transmission schemes.

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