

## Comparison of Analytical Models for Evaluating the Performance of IEEE 802.11 Distributed Coordination Function Under Saturated Conditions

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**Abstract**— There has been huge interest in evaluation of 802.11 networks particularly DCF in recent past among research community. Various analytical models had been proposed for evaluating the performance analysis of 802.11 DCF. These models are based on traffic conditions i.e. either network operates in saturated conditions or non-saturated conditions. In this paper we compare analytical models proposed by Bianchi, Wu et. al. and Chatzimisios et. al. for networks that work under saturated conditions. We have taken throughput as parameter for evaluation purpose. The aim is to acquaint ourselves with mathematical development of these models and redesign these models to provide accurate behavior of IEEE 802.11.

**Keywords**—IEEE 802.11, DCF, saturated traffic, markov process.

### I. INTRODUCTION (HEADING 1)

In recent years wireless networks has gained huge popularity both as product and research work. The 802.11 standard was formulated by IEEE for Wireless Local Area Network [1] which provides the details of medium access control (MAC) and physical layer specifications. The 802.11 standard is based on CSMA/CA algorithm with binary exponential backoff to regulate access to the wireless channel.

In order to access channel, 802.11 supports two mechanisms namely the Distributed Coordination Function (DCF) and Point Coordination Function (PCF). Both of them are based on CSMA/CA techniques. The chief difference between the two can be stated as DCF is random access scheme, which provide asynchronous data transport for accessing the network. While PCF is built on top of DCF to provides centralized MAC support for providing the time bounded services. In our paper we have restricted to DCF access scheme only

DCF uses two techniques for packet transmission. The default one is two-way handshaking mechanism, also called basic access method. For every packet sent from sender station, destination sends the positive acknowledgment (ACK) for successful transmission. In order to overcome the problem of Hidden Stations four way handshaking mechanism, which employs RTS/CTS technique to reserve the channel before transmission is deployed. In last one decade many analytical models have been designed to evaluate the performance of 802.11 DCF.

The analytical models thus designed can be divide into two categories saturated networks and non-saturated networks. In current paper we will be restricting to saturated networks i.e. one in which there is always packets to send in the network. Here we compare the evaluation analysis of various analytical models.

In Section II, approaches to various models along with their assumptions are discussed. Section III deals with mathematical analysis of these approaches leading to throughput calculations in models. Section IV gives insight to the results obtained from these models. Finally, concluding remarks are given in section V.

### II. ANALYTICAL MODELS

Analytical models helps in providing the quantitative analysis of the protocols and models, helping us to predict the result set if the network parameters are changed. There have been few analytical models proposed in late which models [2][3][4][5][6][7][8][9] the operation of DCF in IEEE802.11 networks each with their own set of assumptions.

Bianchi [2] was first to give mathematical model for representing the 802.11 DCF, provided each station is in saturated condition. The main assumptions of this model are: 1. Ideal channel was assumed 2. Collision probability was assumed to be independent of the number of retransmissions. 3. Stations were in saturated condition i.e. they always have packets to transmit 4. Network is homogeneous with finite number of stations 5. There are no hidden stations. IEEE 802.11 network is represented mathematically by two-dimensional discrete-time Markov Chain model. Each station is modeled by two processes ( $s(t)$ ,  $b(t)$ ) where  $s(t)$  is represents backoff stage stochastically and  $b(t)$  is stochastic process representing the backoff time counter at time  $t$ . The model is represented in figure (1). It is assumed that packet can be retransmitted infinitely until successful reception is recorded.

The work in [3] and [4] extends the Bianchi model by considering packet retry limit as mentioned in IEEE802.11 standard. This leads to more realistic and better model. However, other assumptions continue to remain same. The model proposed by both authors produces the same mathematical function for analysis. The paper [4] extends the work of [3] by evaluating the model for other parameters like packet delay also.

In reference [5] by Ziouva an additional state has been introduced in Markov chain model. This additional state represents the case that a station transmits a new packet without ever entering the backoff procedure if it detects that its previous transmitted packet was successfully transmitted and channel is idle. However this is not permitted as per IEEE802.11 standards and leads to unfair situation in network. The result produced also lacks for any validation using simulation.

Since [2] [3] and [4] are most cited works and provide extremely correct results based on their assumptions. We will discuss in detail about the mathematical analysis of these seminal works in next section.

### III. MATHEMATICAL ANALYSIS

According to IEEE802.11 standard before transmitting every station needs to wait for DIFS time. If the station is free for this time period it can begin its transmission. However, if the channel is busy the station needs to defer until the transmission is over. Once the transmission is over station shall start decrementing its backoff counter again until medium is idle. When the backoff counter reaches the value zero and medium is idle the station can proceed towards its transmission. In case of unsuccessful transmission the value of backoff counter is exponential incremented.

Bianchi [2] represented the IEEE802.11 MAC as discrete-time Markov chain model. Since we are assuming saturated conditions, every packet needs to wait for backoff time before being transmitted. The backoff counter is represented by stochastic process  $b(t)$ . The backoff timer is paused when channel is sensed busy, the interval between two consecutive slots can be much longer than slot-time. This leads us to the fact that the value of backoff timer is dependent on its transmission history. Since the Markov is memoryless stochastic process, backoff timer is not sufficient to represent MAC state of the station.

According to IEEE802.11 each station decides the backoff value of as given in equation (1)

$$Backofftime = Random() * aslotime \quad (1)$$

where  $Random()$  is pseudorandom integer drawn from a uniform distribution over the interval  $[0, CW]$ , where  $CW$  is an integer within the range of values of the PHY characteristics  $CW_{min}$  and  $CW_{max}$  equation,  $CW_{min} \leq CW \leq CW_{max}$ . The term  $aslotime$  is the value of the correspondingly named PHY characteristic. The contention window shall take the initial value as  $W=CW_{min}$ . In case of unsuccessful transmission this value increases exponentially as  $CW = 2^i * W$ . The value  $i, i \in (0, m)$ , represent the backoff stage. Let  $m$  be maximum backoff stage such that  $CW_{max} = 2^m * W$ . The backoff stage is represented by stochastic process  $s(t)$  at time  $t$ .

In order to maintain memoryless behavior of the chain it is assumed that collision probability is constant and independent of number of transmission. The term  $p$ ,

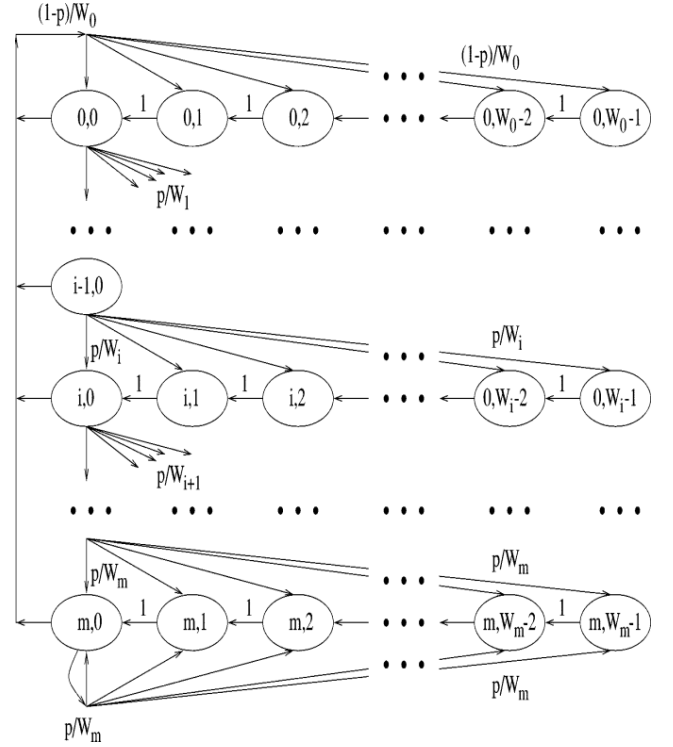


Figure 1. Markov chain for IEEE802.11 designed by Bianchi (source [2])

conditional collision probability, represents collision probability seen by transmitted packet on the channel. This assumption helps us in modeling any station MAC as bi-dimensional process represented by  $(s(t), b(t))$ . Once the Markov chain is in place, taking only non null one step transition probability it is possible to get the closed form solution. The aim of the model is to determine  $\tau$ , probability that a station transmits in a randomly chosen slot. The expression of this probability is represented as:

$$\tau = \frac{2 \cdot (1 - 2p)}{(1 - 2p)(W + 1) + p \cdot W(1 - (2p)^m)} \quad (2)$$

As can be seen from equation (2) the value of  $\tau$  is dependent on the conditional probability  $p$ , which is unknown. Let  $n$  be the number of station in network. The probability that collision occurs is dependent on the fact that one of the remaining  $n-1$  station transmits in given time slot. At steady state the probability  $p$  thus can be represented in equation (3) as

$$p = 1 - (1 - \tau)^{n-1} \quad (3)$$

As can be seen from equation (2) and (3) both are interdependent on each other. This represents a non-linear system and having two unknowns'  $p$  and  $\tau$ . Representing equation number (3) in terms of  $\tau(p)$  we have monotone increasing function. The equation (2) on other hand is

monotone decreasing function. The property of equation (2) and (3) guarantees the unique solution.

Once the value of  $\tau$  is known it is possible to obtain the normalized throughput,  $S$ , for system which is defined as the ratio of the average payload information transmitted in a slot time over the average slot time and represented in equation (4)

$$S = \frac{E[\text{payload information in a slotime}]}{E[\text{Duration of slotime}]} \quad (4)$$

$$= \frac{P_S P_{tr} L}{P_S P_{tr} T_S + P_{tr} (1 - P_S) T_C + (1 - P_{tr}) \sigma}$$

The term  $\sigma$ ,  $T_s$ ,  $T_c$ ,  $L$ ,  $P_s$ ,  $P_{tr}$  represents duration of empty slot time, time required for successful transmission, time when channel is busy after collision, length of payload, probability of successful transmission and probability that there is atleast on transmission on the channel respectively. The values of  $P_s$  and  $P_{tr}$  can be obtained from equation (5) and equation (6)

$$P_{tr} = 1 - (1 - \tau)^n \quad (5)$$

$$P_s = \frac{n\tau(1 - \tau)^{n-1}}{P_{tr}} \quad (6)$$

The value of  $T_s$  and  $T_c$  for basic transmission can be calculated as:

$$T_S = H + L + SIFS + \sigma + ACK + DIFS + \sigma \quad (7)$$

$$T_C = H + L + DIFS + \sigma$$

here the terms  $H$ ,  $L$ ,  $ACK$  and  $\sigma$  are respectively the transmission time needed to send packet header, payload, acknowledgement time and slot time. Detailed analysis of Bianchi model can be referenced in [6].

As assumed in Bianchi model once  $m^{\text{th}}$  stage is reached the packet is not dropped. Rather the packet is being retransmitted until a successful transmission is registered after which the contention window is reset to  $CW_{\min}$ . This assumption is not valid according to IEEE802.11 standard which says each station maintains a retry counter i.e. short retry count and long retry count. Each time packet is unsuccessful transmitted the value of contention is exponential increased leading to increment in the value of retry counter. Once  $CW_{\max}$  value is reached at backoff stage  $m'$  it continues to remain in this state until final value of retry count (backoff stage  $m$ ). On attaining the retry counter threshold the packet is discarded and contention window is reset to  $CW_{\min}$ .

In [3] and [4] this observation was taken into consideration while the rest of assumptions remain same as in [2]. As extension to Bianchi model [2] once maximum backoff,  $m$ , is reached the value of contention is reset to

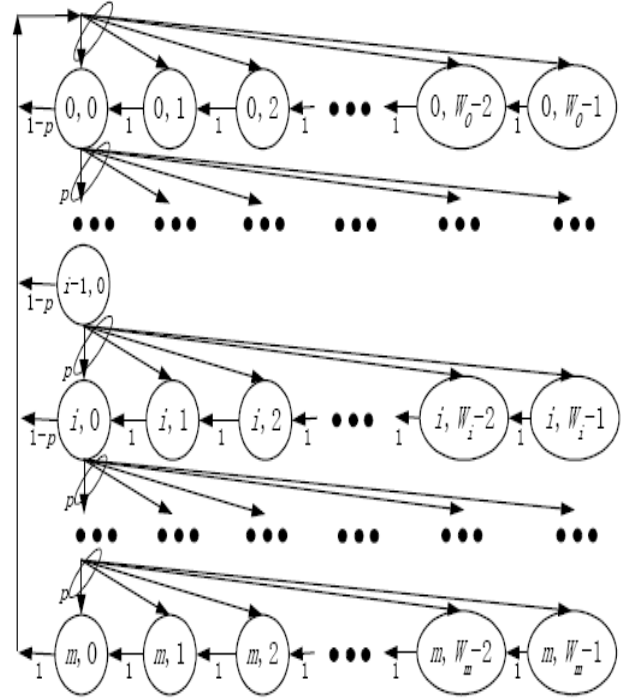


Figure 2. Markov chain for Wu and Chatzimisios models (source [3])

$CW_{\min}$  either due to successful transmission or retry limits is reached. In short, this model considers the effect of retry limit. Mathematically, this fact is represented in equation (8)

$$W_i = 2^i W, i \leq m' \quad (8)$$

$$W_i = 2^{m'} W, i > m'$$

where  $i \in [0, m]$  and  $m$  represents the station retry count.

The discrete time Markov chain is depicted in figure 2. Approaching the problem in fashion similar to Bianchi model discussed above one needs to determine the value of  $\tau$ , probability of transmission in any given empty random slot stated in equation (9)

$$\tau = b_{0,0} \frac{1 - p^{m+1}}{1 - p} \quad (9)$$

where  $b_{0,0}$  can be obtained from equation (10). The other equations including the calculation of collision probability,  $p$ , and saturation throughput,  $S$ , remain same as mentioned in equation (3) to equation (6). Another point of difference is way time for successful transmission,  $T_s$  and collision time  $T_c$  is calculated. Equation (11) and equation (12) shows the computation of these values in [3] and [4] respectively.

$$T_S = H + L + SIFS + \sigma + ACK + DIFS + \sigma \quad (11)$$

$$T_C = H + L + DIFS + SIFS + ACK$$

$$b_{0,0} = \begin{cases} \frac{2(1-2p)(1-p)}{W(1-(2p)^{m+1})(1-p) + (1-2p)(1-p^{m+1})}, & m \leq m' \\ \frac{2(1-2p)(1-p)}{W(1-(2p)^{m'+1})(1-p) + (1-2p)(1-p^{m'+1}) + W \cdot 2^{m'} \cdot p^{m'+1} \cdot (1-2p)(1-p^{m-m'})}, & m > m' \end{cases} \quad (10)$$

$$\begin{aligned} T_s &= H + L + SIFS + ACK + DIFS \\ T_c &= H + L + DIFS + SIFS + ACK \end{aligned} \quad (12)$$

#### IV. RESULTS

The Markov chain models represented in previous section is independent of physical layer and can be applied to any physical standard of IEEE802.11 standard. We have taken throughput as our parameter to compare the analytical results produced by these models. We considered the Direct Spread Sequence Spectrum (DSSS) physical layer used in IEEE802.11b. The other system parameters are specified in Table I.

Our first problem is to solve for the value of  $\tau$ . Utilizing the equation (2) and equation (3) for [2] we can plot the graph. The intersection point of these equations gives us the value of  $\tau$  as specified in figure 3. Similarly we can find out the value of  $\tau$  by plotting equation (3) and equation (10) for [3] and [4]. We have programmed two equations in 'C' language. The resultants are then feed into plotting utility to plot the two equations. We utilized both xgraph [10] and gnuplot [11] for this purpose. The value of  $\tau$  obtained can be substituted in equation (5) and equation (6) to calculate the value of  $P_s$  and  $P_{tr}$ . The values thus obtained can be utilized to get the normalized throughput of network via equation (4).

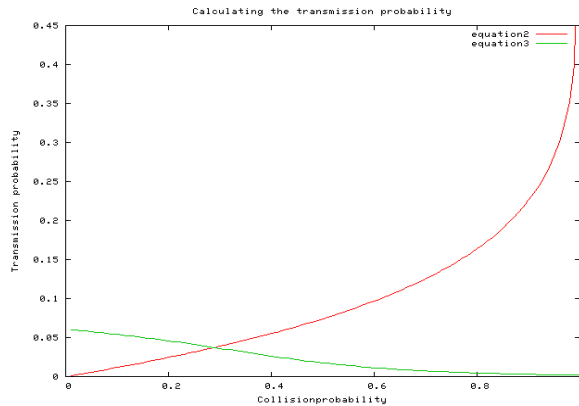


Figure 3. Calculating the value of  $\tau$

For simplicity we have taken only the basic access methods. Figure 4 compares the analytical results of three models verses number of stations. Results shows that as

TABLE I. DSSS SYSTEM PARAMETERS FOR IEEE802.11

System Parameters	Value
Packet Payload	8184 bits
MAC Header	224 bits
PHY Header	192 bits
ACK Header	112 bits + PHY Header
Slot Time	20 $\mu$ s
DIFS	50 $\mu$ s
SIFS	10 $\mu$ s
Propagation Delay	1 $\mu$ s
Channel Rate	1,5.5 and 11Mbits/s
Control Rate	1 Mbits/s
Minimum CW, $W_0$	32
Number of CW Sizes, $m'$	5
Short Retry Limit, $m$	6

the number of contending station goes on increasing the [2] overestimates the throughput as compared to other two models. This difference grows with the number of stations. This fact can be attributes to infinite retransmissions of packet in  $m^{\text{th}}$  stage until the success is registered. While results obtained for the model given by [3] and [4] are same.

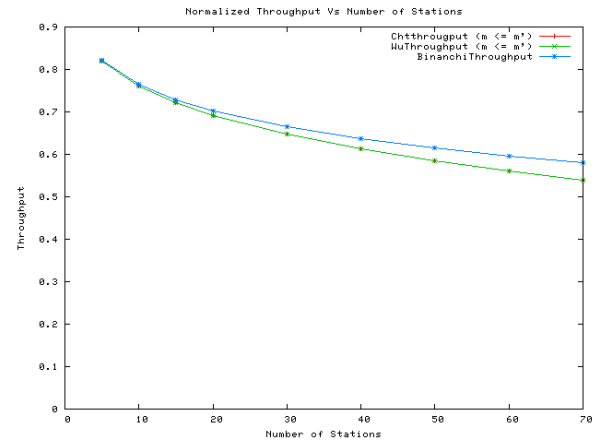


Figure 4. Throughput Efficiency fo Basic Rate when  $m \leq m'$

An interesting fact is observed when we considers the case  $m > m'$  i.e. when the maximum contention window ( $m'$ ) is reached, its value continues to be  $CW_{\text{max}}$  until the short retry values ( $m$ ) is obtained. The throughput obtained for all three models are almost equal as can be seen in equation (5).

Keeping the number of stations as fixed we increase the size of contention window to study the effect on throughput for three models. As can be seen from figure 6

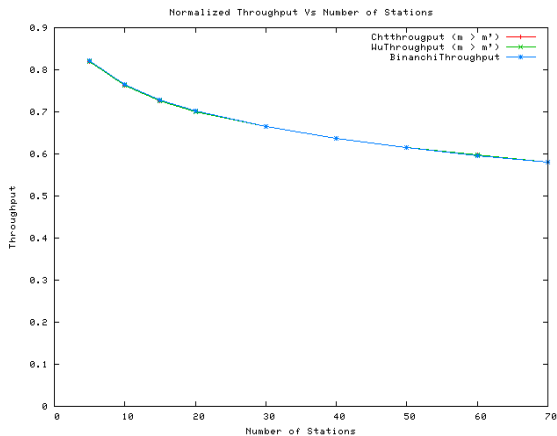


Figure 5. Throughput Efficiency fo Basic Rate when  $m > m'$

the throughput obtained from Bianchi model is higher as compated to other models. But this difference diminishes as the size of contention window is increased exponentially. While the result, figure7, for three models once we have reached the maximum contention window ( $m'$ ) until the packet in transmission will be dropped ( $m$ ) shows similar results. It can be seen that the throughput of network increases when the size of initial contention window is increased.

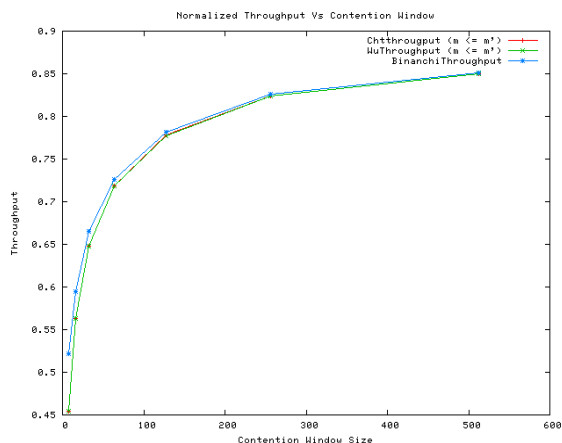


Figure 6. Throughput analysis when considering basic rate for  $m \leq m'$

## V. CONCLUSION

This paper presents the analysis of three mathematical models representing IEEE802.11 DCF as Markov chain. The analysis is not exhaustive. Instead, it mainly focuses on the mathematical concepts involved in these analytical models. It can be seen that assuming all ideal conditions Bianchi model tends to overestimate the throughput in comparison to other models which considers discarding

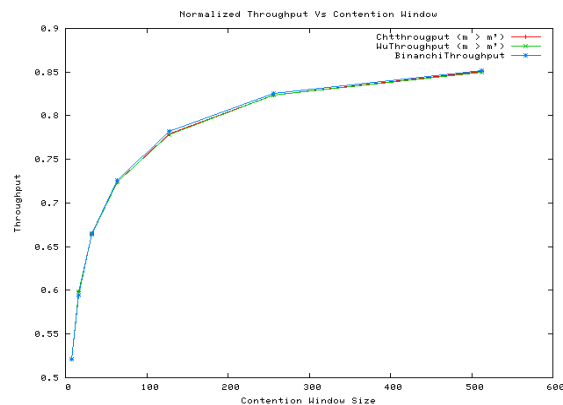


Figure 7. Throughput Efficiency fo Basic Rate when  $m > m'$

the packets once short retry limit is reached. Although many analytical models has been proposed with their own set of assumptions. Researchers are required to develop the model considering more real conditions for IEEE802.11 networks.

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