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Review of Mechanical Modelling of Fixed-Fixed Beams in RF MEMS Switches

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***Abstract*— This paper addresses mathematical analysis of the mechanics involved in RF MEMS Switch. The fixed- fixed beam undergoes deflection in y direction when a force is applied onto the beam. Fixed-Fixed beams are used due to there ease of high spring constant and ease of manufacturing. The first section of paper deals with mechanical analysis of the beam i.e. derivation of reaction forces and reaction moment. The later section of paper deals with the derivation of spring constant of fixed-fixed beam. Along with the fast development of the automotive electronics technology, the wireless communication technique on vehicles has huge market potential. This paper takes the cantilever beam MEMS switch as an example to discuss mechanical principle and function simulation of the RF MEMS switch used in Vehicle-carried radio frequency communication. The increasing demand for low loss, high Q devices for high frequency applications has led to the development of MEMS components in the RF domain. RF MEMS has evolved over the past decade and it has emerged as a potential technology for wireless, mobile and satellite communication and defence applications. Extensive research has been carried out to identify and overcome the limitations of RF MEMS technology for replacing PIN or FET based switches for low- loss applications. The main advantage of this technology is that the devices can be manufactured by processes similar to that of VLSI and the advancement of VLSI technology has helped in the realization of many sub millimeter- sized parts to provide RF functionality.**

The spring constant for fixed- fixed beam is modelled in two parts – One due to stiffness of bridge which accounts for material properties and other due to residual stress. This paper deals with the spring constant due to stiffness of bridge .A comparative study of various materials with different young's modulus is also carried out.

***Keywords*— Fixed Fixed beam, Vertical reaction, moment Equation, Spring constant,Distributed load,Youngs Modulus,Vertical deflection.**

I.Introduction

Microelectromechanical Systems (MEMS) have been developed since 1970s for various types of sensors, switches; accelerometers etc. RF MEMS switches are miniature devices that use mechanical movement to achieve a short and open circuit in transmission line. This paper presents as study of Fixed-fixed beams used in RF MEMS switches. [1] As a key circuit element in the field of microwave control, RF MEMS switch has wide applications in phase arrays and configurable apertures for defense and telecommunication systems, switching networks for satellite communication.
RF MEMS switches have several advantages over PIN diode and FET switches such as low insertion loss, high isolation, excellent compatibility with current microwave and mm-wave circuit. Because of those significant advantages mentioned above, RF MEMS switch is increasingly under focus [2][3].

II. Fixed-Fixed Beam

The mechanical operation of RF MEMS switches starts with analysis of Fixed-Fixed beam. The mechanical behaviour of beam can be modelled using a spring constant k(N/m).The deflection δ(Ym), of fixed-fixed beam for an external force F(N),can be obtained using the formula F=k δY.

This paper throws light on mathematical evaluation of reaction forces when a force F(N) is applied to fixed–fixed beam. When a concentrated vertical load F(N) is applied onto the beam there is a vertical deflection δY. Since beam is hinged at two ends i.e beam is a fixed–fixed there is no horizontal deflection in beam when force is applied.

Fixed –Fixed beam of length L (μm) is placed in X-Y plane and a concentrated load F(N) is applied onto it.

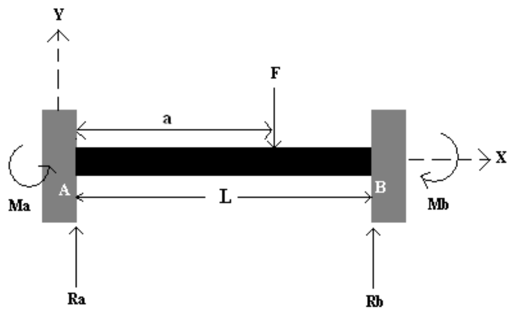

Figure 1.Fixed-Fixed Beam with concentrated load.

R_a: Vertical Reaction at left end.
R_b: Vertical Reaction at Right end.
M_a: Reaction Moment at left end.
M_b: Reaction Moment at right end.
F: Force applied on beam.

The deflection versus load position is given by [4].

$$\mathbf{EI\ d^2y/dx^2 = M_a + R_a x} \quad (1.1)$$

Where E is the Young's Modulus, I is the Moment of Inertia, M_a is the moment at left end and R_a is the vertical reaction at left end. The above equation is also known as Moment Equation.

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To evaluate the vertical reaction and moment forces take a section at a distance of x from left end of the beam as shown in the Fig 1.1.

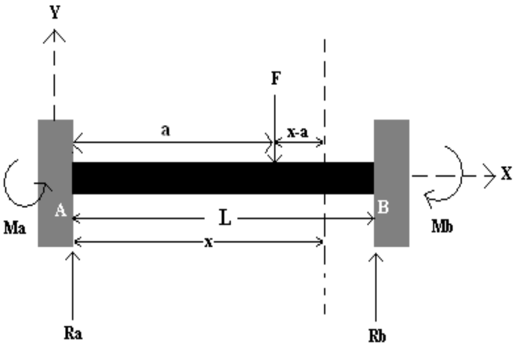

Figure 2. .Fixed-Fixed Beam with concentrate load.

Rewriting the Eq 1.1 with reference to the section taken at a distance x, the moment equation becomes

$$\mathbf{EI\ d^2y/dx^2 = R_a x - F(x-a) - M_a} \qquad (1.2)$$

where the moment due to force F; M_a are in anti-clockwise direction and moment due to reaction force Ra is in clockwise direction.

Integrating the above equation with respect to x we get;

$$EI\frac{dy}{dx} = \frac{R_a x^2}{2} - \frac{F(x-a)^2}{2} - M_a x + c \qquad (1.3)$$

At x=0 ;dy/dx=0 i.e. slope at ends is zero.
Substituting in 1.3 we get c=0.

$$EI\frac{dy}{dx} = \frac{R_a x^2}{2} - \frac{F(x-a)^2}{2} - M_a x \qquad (1.4)$$

on integrating (1.4) w.r.t x we get ;

$$EIy = \frac{R_a x^3}{6} - \frac{F(x-a)^3}{6} - \frac{M_a x^2}{2} \qquad (1.5)$$

Since beam is fixed at both ends the deflection is zero at both ends of beam i.e. y=0 at x=L.

$$0 = \frac{R_a L^3}{6} - \frac{F(L-a)^3}{6} - \frac{M_a L^2}{2}$$

Rearranging the above equation we get;

$$\mathbf{M_a = \frac{R_a L}{3} - \frac{F(L-a)^3}{3L^2}} \qquad (1.6)$$

Also the slope at the end of beam is zero i.e. dy/dx=0 at x=L.Substituting the above mentioned condition in equation (1.4) we get;

$$EI(0) = \frac{R_a L^2}{2} - \frac{F(L-a)^2}{2} - M_a L$$

Substituting M_a from (1.6) we get;

$$0 = \frac{R_a L^2}{2} - \frac{F(L-a)^2}{2} - \left[\frac{R_a L}{3} - \frac{F(L-a)^3}{3L^2}\right] * L$$

$$Ra\left[\frac{L^2}{3} - \frac{L^2}{2}\right] = \frac{F(L-a)^3}{3L} - \frac{F(L-a)^2}{2}$$

$$\mathbf{Ra = \frac{F(L-a)^2 (L+2a)}{L^3}} \qquad (1.7)$$

Substituting Ra in (1.6) the expression for M_a can be evaluated as;

$$\mathbf{M_a = -\frac{F(L-a)^2}{L^2}}$$

The above mathematical analysis calculates the value for Vertical reaction force and moment.

The above derived equations of Vertical reaction Ra and moment M_a are plotted against the applied force F(N), a(µm).

The 3-D plot shows variation of reaction Ra in accordance to force and the distance at which it is applied. The plotting is done using Matlab 7.0.

III.3-D Plot of Vertical Reaction (Ra)

The vertical reaction is plotted with respect to the applied force F and distance a, are varied simultaneously and the combined effect due to force and variation in distance is plotted. The beam is taken of a length of 300 um.The plotting is carried out using the meshgrid function in Matlab7.0 [6].

Matlab Code for the Vertical Reaction equation is given as below:

```
l = 300*10^(-6);
x = linspace(0,40,100);
y = linspace(0,300*10^(-6),100);


[F,a] = meshgrid(x,y);
Ra = F.*((l-a).^2)*(l+(2.*a))/(l^3);
%subplot(2,2,1);
surfc(F,a,Ra);

view(-55,40);
title('Reaction force')

xlabel('F(N)')
ylabel('a(um)')
zlabel('Ra(N)')
```

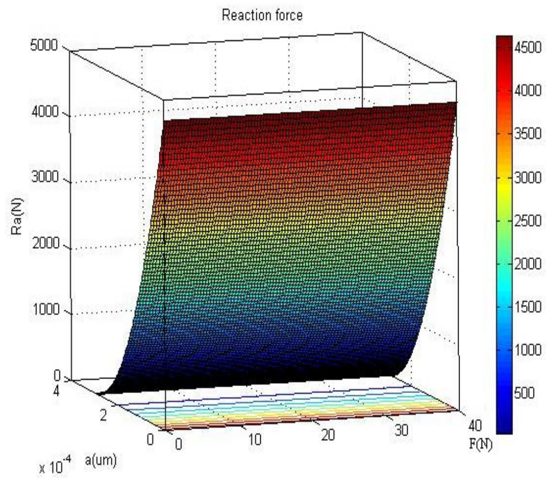

Figure 3.D Plot showing reaction force in accordance with a (µm) and F (N).

The X axis plots Force (N), Y axis represents a (µm) and Z axis gives the vertical reaction (R_a).

IV.3-D Plot of Reaction Moment (Ma)

The plot of Moment is also studied using Matlab Code for the Reaction Moment equation is given as below:

```
l = 300*10^ (-6);
x = linspace(0,40,100);
y = linspace(0,300*10^(-6),100);
[F,a] = meshgrid(x,y);

Ma = F.*a.*((l-a). ^2)/ (l^2);
%subplot(2,2,3);
figure(3);
meshc(F,a,Ma);
view(-55,40);
title('Moment')
xlabel('F(N)')
ylabel('a(um)')
zlabel('Ma(N.m)')
```

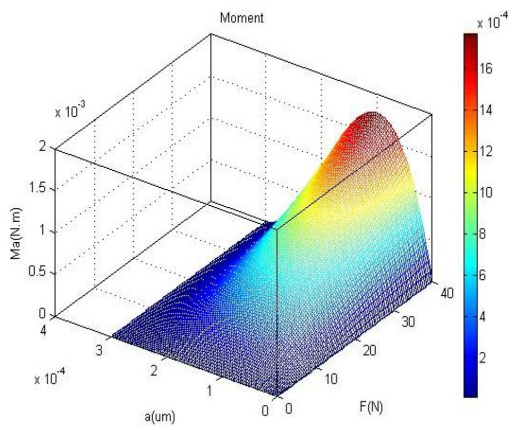

Figure 4. 3D Plot showing reaction force in accordance with a (µm) and F (N).

The X axis plots Force (N), Y axis represents a (µm) and Z axis gives the vertical reaction (R_a).

V.Spring Constant for Distributed load

For a distributed load across the entire beam the deflection y is given as [1] [6] ;

$$Y = \frac{2}{EI} \int_{l/2}^{l} \frac{\xi}{48} (l^3 - 6l^2a + 9la^2 - 4a^3)\, da \qquad (1.8)$$

Where ξ is the load per unit length so that total load becomes F= ξ l.Since the load is symmetric the integral is evaluated from l/2 to l and multiplied by 2.

$$Y = \frac{2\xi}{48EI} \left[(l^3a - 3l^2a^2 + 3la^3 - a^4)\right]_{l/2}^{l}$$

Putting the limits we get;

$$Y = 2 * \frac{2\xi}{48EI} [-l^4 / 32]$$

Moment of inertia I is given by $wt^3/12$, where w is the width and t is thickness of beam, so y becomes;

$$Y = \frac{\xi}{E\,wt^3} * \frac{[-l^4]}{32} \;;$$

We know that force F= -K δY where K is the spring constant due to stiffness of beam.

$$K = -F/y = -\frac{\xi l * E\,wt^3\ 32}{\xi * [-l^4]}$$

$$\mathbf{K = 32\,Ew\,(t/l)^3}$$

VI. Spring Constant of Fixed Fixed beam with different Young's Modulus

A fixed fixed beam of length 300 µm with thickness of 1 µm is taken and K/w is plotted versus t/l.Four different materials gold,aluminium,copper and platinum with young modulus(E)80Gpa,69Gpa,130Gp,168Gpa[5],[6] respectively are taken.

The plotting is carried out in Matlab considering force to be distributed over the entire beam.

Matlab Coding for Evaluating Spring Constant of Different Beams

```
t=linspace(0,1*10^(-6),200);
l=300*10^ (-6);
tbyl=t./l;
%tbyl=linspace(10,20,50);
kbyw=0:0.2

E=80*10^9;
kbyw=32*E*(tbyl.^3)*10^(-6);
Subplot (2,2,1);
```

```
plot (tbyl,kbyw)
title ('spring constant for gold beam')
xlabel('t/l')
ylabel('k/w')

E=69*10^9;
kbyw=32*E*(tbyl.^3)*10^(-6);
subplot (2,2,2);
%figure (2)
plot (tbyl,kbyw)
title('spring constant for aluminium beam')
xlabel('t/l')
ylabel('k/w')

E=130*10^9;
kbyw=32*E*(tbyl.^3)*10^(-6);
subplot (2,2,3);
%figure (3)
plot (tbyl,kbyw)
title ('spring constant for copper beam')
xlabel('t/l')
ylabel('k/w')

E=168*10^9;
kbyw=32*E*(tbyl.^3)*10^(-6);
subplot (2,2,4);
%figure (4)
plot (tbyl,kbyw)
title ('spring constant for platinum beam')
xlabel ('t/l')
ylabel('k/w')
```

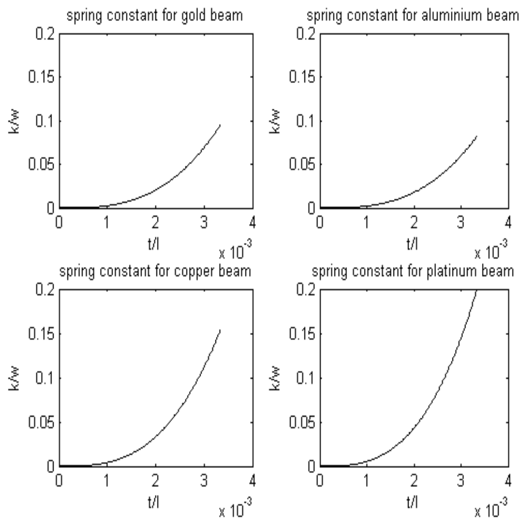

Figure 5. Spring Constant (with respect to beam width w) versus t/l of different beams where force is distributed over entire beam.

VII. Conclusions

This paper introduced mathematical analysis of the mechanics involved in RF MEMS switch. The paper has been presented in two sections. First section addressed the derivation of reaction force and reaction moment associated with the fixed-fixed beam used in MEMS. The later addressed the derivation of the spring constant and variations in it with respect to different materials.

Mathematical analysis of both the sections are done using the Matlab software including 3-D analysis of first section and 2-D analysis of the later. The Matlab results concluded our objectives and completes the analysis of mechanics involved in Fixed –Fixed beam of RF MEMS switches.

VIII.References

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