To generate convincingly rendered images of surface reflection appearances, computer graphics usually relies on bidirectional reflectance distribution function (BRDF) models. One such model is the Phong reflectance model (see the “Related Work” sidebar). In believable rendering applications, these intuitive models don’t have to be physically accurate as long as the resulting image satisfies the viewer’s expectations. For example, the traditional Phong model is not energy conserving. However, predictive rendering, which we focus on here, relies on BRDF models to make valid assumptions of the surface’s physical behavior to correctly describe the light transport in complex scenes. Two well-known examples of physically plausible BRDF models are the Fresnel reflectance and Torrance-Sparrow models (see the “Related Work” sidebar). They describe the surface reflectance behavior down to its reflective polarization properties.

To verify the accuracy of these physically based models, we collected a set of metallic surfaces. Then, we used an ellipsometer to measure the polarization properties of their reflected light. Finally, we compared the measurements to the reflectances we predicted from the physically plausible BRDF models.

**Why Use Ellipsometry?**
First, in physics, ellipsometry is a standardized process for measuring surface properties of objects (for example, surface layer thickness). It comes with off-the-shelf measurement equipment sufficient for our needs. Thus, we don’t need to engineer and carefully calibrate a suitable gonioreflectometer (a device that measures BRDFs) on our own.

Second, for BRDF models that try to model reflectance polarization, we want to measure their correctness on the basis of the quality of reflective polarization prediction. Our main argument is that the correct prediction of the polarization state implies reflectance value prediction.

**Our Measurement Approach**
In computer graphics, reflected light is usually represented as a triplet of RGB values (see Figure 1a) and measured with a gonioreflectometer for varying incident and exitant angles. Normally, we would either store the measured intensity values in lookup tables or approximate them with BRDF models that can be quickly evaluated on a GPU in real time.
In our approach we compute BRDF values from measured polarizing behaviors. We choose this approach because we don’t consider light as a scalar value. Instead, we think of it as an electromagnetic wave traveling through space with a field behavior in space and time. The wave is described by its parallel (p-plane) and perpendicular (s-plane) components.

In Figure 1b, the incident wave $\vec{E}_\lambda$ with a wavelength $\lambda$ is reflected from the surface to become the reflective wave $\vec{R}_\lambda$. Here, a linearly polarized light wave $\vec{E}_i$ is reflected as an elliptically polarized wave $\vec{R}_i$. (In this image, $\vec{E}_i$ represents the incident wave.) We measure the wavelength-dependent ratio of a surface’s parallel and perpendicular reflectance values with an ellipsometer. The combination of both values defines the reflected light’s perceived intensity. The components of $\vec{E}_i$ and $\vec{R}_i$ are given in the s- and p-plane.

**The Fundamental Equation of Ellipsometry**

Using the ellipsometer, we first measure the reflected light’s polarization state for a varying wavelength $\lambda$. The state is characterized by $\Psi$ and $\Delta$, which are the amplitude attenuation and phase shift induced on the reflected light wave. From these values, we want to compute the ratio of $r_p$ and $r_s$ values, against which we compare the existing physically plausible models’ estimated reflectance values.

For this conversion, we use the fundamental equation of ellipsometry:

$$\rho = \tan(\Psi)^{\Delta} = \frac{r_p}{r_s}, \quad (1)$$

where $\rho$, $r_p$, and $r_s$ are elements of C. (The related sidebar provides a shortened derivation of the equation.) Because our ellipsometer can measure $\Psi$ and $\Delta$ only for a given angle of inclination $\theta$ and wavelength $\lambda$, we compute $\rho_{\text{data}} = \tan(\Psi)^{\Delta}$ from the measured $\Psi$ and $\Delta$ values, using Equation 1. We then can compare the models to these values.

Figure 1. Two views of reflected light. (a) In computer graphics, reflected light is usually considered as a scalar or as an RGB triplet. In this image, $I_{\text{rgb}}$ represents the incident light’s intensity and $R_{\text{rgb}}$ represents the exitant light’s intensity. (b) Instead, we consider the spectrum of light waves $\vec{R}_\lambda$, which are attenuated and shifted in phase. Here, a linearly polarized light wave $\vec{E}_i$ is reflected as an elliptically polarized wave $\vec{R}_i$. (In this image, $\vec{E}_i$ represents the incident wave.) We measure the wavelength-dependent ratio of a surface’s parallel and perpendicular reflectance values with an ellipsometer. The combination of both values defines the reflected light’s perceived intensity. The components of $\vec{E}_i$ and $\vec{R}_i$ are given in the s- and p-plane.

Figure 2. We evaluated (a) steel, copper, galvanized steel, and anodized aluminum and (b) a gold and a silver coin. We used the samples to measure the polarization states. We later compared the measured values to the polarization reflectance predicted by the bidirectional reflectance distribution function (BRDF) models.

Then, we only have to derive $r_{\text{model}}$ and $r_{\text{model}}$ for a BRDF model that is considered physically plausible:

$$\rho_{\text{model}} = \frac{r_{\text{model}}}{r_{\text{model}}}.$$

Finally, we fit $\|\rho_{\text{model}}\|^2$ for the model parameters to the values $\|\rho_{\text{data}}\|^2$ we just computed from the measured $\Psi$ and $\Delta$ values. We assume a complex index of refraction with $n$, $k > 0$. 
Related Work in BRDF Models and Ellipsometry

Bui Tuong Phong introduced the intuitive modeling of surfaces’ light reflection. He constrained the model only to the principles of reciprocity and positivity. Gregory Ward and his colleagues measured the reflection process and fitted a suitable function to the numerical results. Wojciech Matusik and his colleagues captured a large database of only the goniometric reflectance values for a huge variety of materials. However, they didn’t provide a model to approximate the values.

For physically based rendering, we instead need plausible bidirectional reflectance distribution function (BRDF) models. These models assume an accurate representation of idealized surface patches. In the physics community, Kenneth Torrance and Ephraim Sparrow introduced the most prominent model in 1967. Many researchers have adapted it; for example, James Blinn and Robert Cook adapted it to use different microfacet models. However, these models haven’t been experimentally verified down to their polarization behavior. Researchers have recently addressed polarization-aware surface reflectometry but have presented measured data instead of verifying existing models. Our evaluation (see the main article) focuses on Torrance and Sparrow’s BRDF model as a representative of physically plausible models, and we vary it over different microfacet models.

Also, we compare our real-world measurements to the Fresnel equations. We seek to verify physically plausible models with ellipsometry equations. The equations, as we use them, were derived by Paul Drude in 1887. Around 1901, he depicted an ellipsometric-measurement instrument (see Figure A1). Actually, Jules-Celestin Jamin is believed to have already invented such an instrument in 1847. His instrument consisted of two telescopes with attached linear polarizers, and the image was formed at the naked human eye. But such devices weren’t called ellipsometers until 1945, when Alexandre Rother introduced the term.

Until the 1970s, ellipsometry didn’t take wavelength dependency into account. The technique was to rotate the analyzer until you reached an intensity minimum—ideally, zero intensity (null ellipsometry). This method was time-consuming; measuring a surface patch took approximately one hour.

So, the photometric ellipsometer emerged. The polarizer is fixed at 45 degrees, and a retarder is optionally placed in the beam. The reflected light is always elliptically polarized, so that the analyzer simply has to rotate continuously; the resulting beam’s intensity is a sinusoid over time. After Fourier analysis, you can retrieve the amplitude attenuation and phase shift induced on the reflected light wave.

Our ellipsometer is principally based on the spectroscopic-ellipsometer configuration that David Aspnes and Ambrose Studna developed in 1975. A white light source produces the emitted light, and the retrieved light passes a monochromator (see Figure A2).

![Figure A. Ellipsometers. (1) Jules-Celestin Jamin introduced the earliest ellipsometer, which measures the reflection polarization for materials. A surface S is placed between telescopes F and K, both attached to polarizers p and p’ (back then, realized with Nicol prisms). C represents the Babinet-Soleil compensator, a retarder. The image is formed in the human eye. (Source: Lehrbuch der Optik, out of copyright.) (2) We used the Sentech ellipsometer, which implements spectroscopic ellipsometry. A white light source produces the emitted light, and the retrieved light passes a monochromator.](image)

The Samples
We chose copper, steel, galvanized steel, aluminum, gold, and silver (see Figure 2). We performed measurements for both commodity aluminum foil and anodized aluminum, which is usually sold in hardware stores. We used a freshly minted gold coin and silver coin with 99.9 percent purity. We did not further clean the coins, so we did not introduce a film layer before measurement.

The Ellipsometer
We used the Sentech SE-800 ellipsometer (see Fig-
 Recent approaches to digitizing cultural heritage have also employed spectroscopic ellipsometry.\textsuperscript{10,11} However, they only reconstructed artifacts made of bronze and copper. In our research, we examined copper, steel, aluminum, gold, and silver.

**References**


We evaluated $\Psi(\lambda)$ and $\Delta(\lambda)$ for a finite set of angles $\theta_i$, using the ellipsometer’s goniometer. We limited the measurements to six incident angles: 45, 50, 60, 70, 75, and 80 degrees. The incident angle ($\theta_i$) and exitant angle ($\theta_o$) were always identical.

**Results**

We evaluated the data in the visible spectrum: 380 to 750 nm. Then, we computed the reflectance ratios for varying wavelengths using Equation 1, and we checked whether the computed values fit the values predicted in the literature.\textsuperscript{2} Finally, we fitted the physically based BRDF models—that is, the Torrance-Sparrow model with different distribution functions—to get a statement about the goodness of their applicability to real-world surface materials.

Copper

From the measured $\Psi$ and $\Delta$ values (see Figure 4a), we computed the squared reflectance ratio $|\tilde{r}_{data}|^2 = \left| \tan(\Psi) e^{i\Delta} \right|^2$ and compared it to the estimated reflectance ratio from the literature (see Figure 4b).

Then, we performed the fitting. We examined the Fresnel terms

![Diagram of the ellipsometer](image-url)
Material Appearance

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{rgb}$</td>
<td>Incident-light radiance.</td>
</tr>
<tr>
<td>$R_{rgb}$</td>
<td>Exitant-light radiance.</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Wavelength of light.</td>
</tr>
<tr>
<td>$E_i$</td>
<td>Incidence wave.</td>
</tr>
<tr>
<td>$R_i$</td>
<td>Reflective wave.</td>
</tr>
<tr>
<td>$r_p$</td>
<td>Parallel field component of the reflected light wave with regard to the plane of incidence. $r_p \in \mathbb{C}$.</td>
</tr>
<tr>
<td>$r_s$</td>
<td>Perpendicular field component of the reflected light wave with regard to the plane of incidence. $r_s \in \mathbb{C}$.</td>
</tr>
<tr>
<td>$\psi$, $\tan \psi$</td>
<td>Attenuation of light induced by surface reflectance. Wavelength dependent.</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>Phase shift of light induced by surface reflectance. Wavelength dependent.</td>
</tr>
<tr>
<td>$\rho_{\text{model, data}}$</td>
<td>Complex reflectance ratio. Estimated with a bidirectional reflectance distribution function (BRDF) model or computed from $\psi$ and $\Delta$. Wavelength dependent. $\rho \in \mathbb{C}$.</td>
</tr>
<tr>
<td>$\theta$, $\theta_i$, $\theta_o$</td>
<td>Inclination angle with regard to the surface normal. $\theta \in (0^\circ \ldots 90^\circ)$. In this article, the incident angle $\theta_i$ equals the exitant angle $\theta_o$.</td>
</tr>
<tr>
<td>$F_{\text{FL,FL}}(\lambda)$</td>
<td>Fresnel reflection terms for parallel and perpendicular polarized light. Wavelength dependent.</td>
</tr>
<tr>
<td>$n$</td>
<td>Refractive index of the surface material. Wavelength dependent.</td>
</tr>
<tr>
<td>$k$</td>
<td>Absorption coefficient of the surface material. Wavelength dependent.</td>
</tr>
<tr>
<td>$G$</td>
<td>Geometry term that models the shadowing and masking.</td>
</tr>
<tr>
<td>$D_n$</td>
<td>Microfacet distribution function. In this article, $n$ varies over the Blinn-Phong, Gaussian, Beckmann, and Trowbridge-and-Reitz distributions.</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Surface roughness expressed as the root-mean-square slope. $\beta \in (0^\circ \ldots 90^\circ)$</td>
</tr>
<tr>
<td>$\overrightarrow{I}$</td>
<td>Incoming direction.</td>
</tr>
<tr>
<td>$\overrightarrow{V}$</td>
<td>Outgoing direction.</td>
</tr>
<tr>
<td>$\overrightarrow{N}$</td>
<td>Microfacet normal.</td>
</tr>
<tr>
<td>$\overrightarrow{n}$</td>
<td>Surface normal.</td>
</tr>
</tbody>
</table>

$$\eta = \frac{1}{4\cos(\theta_i)\cos(\theta_o)} * Fr \ast G(\theta_i, \theta_o) * D_n,$$

with the geometric term $G$ for shadowing illuminating rays and masking absorbed viewing rays and where $i \in \{\perp, \parallel\}$. In our measurement setup, $\theta_i$ equals $\theta_o$ and in the fitting procedure, $G$ is constant and $n$ varies over these distributions:

- Blinn-Phong,
- Gaussian,
- Beckmann, and
- Trowbridge and Reitz.

More information about these distributions and their formulas can be found in the sidebar “Physically Plausible BRDFs and Microfacets.”

We used Mathematica’s `NonLinearModelFit` function and provided an initial guess for $n$ and $k$ and for the surface roughness $\beta$ in the microfacet distribution for the given $\lambda$. Figure 4c plots the fitting results for $n$ and $k$ in the visible spectrum against the predicted values. The predicted values slightly exceed those for the fitted $k$.

Table 1 lists the detailed fitting results for copper. We performed the fitting for steel and galvanized steel in the same way; the table also shows those results.

Aluminum

For anodized aluminum, the measured data showed interesting behavior. The plots for $\psi$ and $\Delta$ oscillate over the measured spectrum (see Figure 5a). Consequently, the computed reflectance ratio also oscillates over that spectrum (see Figure 5b).

A fitting such as we did for copper couldn’t be performed ad hoc. This was probably due to an aluminum oxide layer on top of the aluminum surface. We assume a thin, porous layer of 3 $\mu$m on top of the substrate with a known $n$. For the substrate, we assumed $n = 0.11945$ and $k = 2.26534$ for $\lambda = 550$ nm. When we examined aluminum foil with this much thinner oxide layer, we found that the computed reflectance ratio values approximately fit the predicted behavior (see Figure 5c). Table 1 lists the results.

Gold and Silver

From the measured $\psi$ and $\Delta$ values for the gold coin (see Figure 6a), we computed the reflectance ratio $\left|\mathbf{\eta_{data}}\right| = \left|\tan(\psi) e^{\Delta i}\right|$ and compared it to the reflectance ratio in the literature (see Figure 6b). The measured data fit accurately with the predicted values. As with copper, we could fit $n$ and $k$ to the measured data points (see Figure 6c).

We could also fit $n$ and $k$ for the silver coin (see...
Figure 7). For nonpolished surface spots, the values in the literature exceed those for \( k \) and slightly exceed those for \( n \). If the coin is a proof, the reflectance ratio drops noticeably for wavelengths beyond 470 nm; this leaves space for further research.

Table 1 lists the results for gold and silver.

Table 1

<table>
<thead>
<tr>
<th>( n )</th>
<th>( k )</th>
</tr>
</thead>
<tbody>
<tr>
<td>literature</td>
<td>measured</td>
</tr>
<tr>
<td>gold</td>
<td>silver</td>
</tr>
</tbody>
</table>

Finally, we compared the renderings generated with the fitted \( n \) and \( k \) to those generated with the literature values. Using the state-of-the-art renderer PBRT 2.0, we modeled a scene showcasing a half-sphere illuminated by the Uffizi light probe. For the half-sphere, we set \( \beta = 0.0001 \) to increase the reflection of the surrounding scene.

Figure 8 shows the renderings. Figure 8a shows the sphere's surface with \( n \) and \( k \) fitted to our ellipsometric measurement data (compare with Figures 4c, 6c, and 7c). Figure 8b shows the sphere's surface as...
Figure 5. The results for aluminum. (a) Anodized aluminum shows oscillation in the visible spectrum. Blue indicates the \( \Psi \) values, and red indicates the \( \Delta \) values, for an inclination of \( \theta = 60 \) degrees. (b) The reflectance ratios \( r_p/r_s \) computed with Equation 1 (the blue dots) for varying \( \theta \) and \( \lambda \). The results don’t match the reflectance ratios \( F_r/F_{\parallel} \) of aluminum as stated in the literature\(^2\) (the black grid). (c) However, aluminum foil shows the expected behavior in the measured range.

Table 1. The results for fitting the Fresnel terms and Torrance-Sparrow bidirectional reflectance distribution function (BRDF) with different microfacet distributions for the examined metals.

<table>
<thead>
<tr>
<th>Model</th>
<th>Parameter(^*)</th>
<th>Copper, ( \lambda = 516 ) nm</th>
<th>Galvanized steel, ( \lambda = 500 ) nm</th>
<th>Steel, ( \lambda = 500 ) nm</th>
<th>Aluminum foil, ( \lambda = 516 ) nm</th>
<th>Gold, ( \lambda = 516 ) nm</th>
<th>Silver, ( \lambda = 495 ) nm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fresnel terms</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( n )</td>
<td>1.030</td>
<td>3.708</td>
<td>2.400</td>
<td>0.875</td>
<td>0.549</td>
<td>0.290</td>
<td></td>
</tr>
<tr>
<td>( k )</td>
<td>2.460</td>
<td>4.721</td>
<td>3.699</td>
<td>6.233</td>
<td>1.810</td>
<td>3.040</td>
<td></td>
</tr>
<tr>
<td>Torrance-Sparrow BRDF with Blinn-Phong distribution</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( n )</td>
<td>1.032</td>
<td>3.708</td>
<td>2.400</td>
<td>0.875</td>
<td>0.549</td>
<td>0.289</td>
<td></td>
</tr>
<tr>
<td>( k )</td>
<td>2.459</td>
<td>4.721</td>
<td>3.700</td>
<td>6.233</td>
<td>1.810</td>
<td>3.040</td>
<td></td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.786</td>
<td>0.867</td>
<td>0.255</td>
<td>0.637</td>
<td>0.443</td>
<td>0.793</td>
<td></td>
</tr>
<tr>
<td>Torrance-Sparrow BRDF with Gaussian distribution</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>( n )</td>
<td>1.232</td>
<td>3.708</td>
<td>2.471</td>
<td>0.876</td>
<td>0.550</td>
<td>0.289</td>
<td></td>
</tr>
<tr>
<td>( k )</td>
<td>2.458</td>
<td>4.721</td>
<td>3.500</td>
<td>6.233</td>
<td>1.810</td>
<td>3.040</td>
<td></td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.358</td>
<td>0.213</td>
<td>0.307</td>
<td>0.438</td>
<td>0.218</td>
<td>0.825</td>
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<td>Torrance-Sparrow BRDF with Beckmann distribution</td>
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<tr>
<td>( n )</td>
<td>1.032</td>
<td>3.708</td>
<td>2.400</td>
<td>0.875</td>
<td>0.549</td>
<td>0.289</td>
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<tr>
<td>( k )</td>
<td>2.459</td>
<td>4.721</td>
<td>3.700</td>
<td>6.233</td>
<td>1.810</td>
<td>3.040</td>
<td></td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.786</td>
<td>0.867</td>
<td>1.180</td>
<td>0.637</td>
<td>0.866</td>
<td>0.793</td>
<td></td>
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<tr>
<td>Torrance-Sparrow BRDF with Trowbridge-and-Reitz distribution</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>( n )</td>
<td>1.232</td>
<td>3.708</td>
<td>2.471</td>
<td>0.875</td>
<td>0.550</td>
<td>0.289</td>
<td></td>
</tr>
<tr>
<td>( k )</td>
<td>2.458</td>
<td>4.721</td>
<td>3.500</td>
<td>6.233</td>
<td>1.810</td>
<td>3.040</td>
<td></td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.187</td>
<td>0.259</td>
<td>0.246</td>
<td>0.501</td>
<td>0.185</td>
<td>0.736</td>
<td></td>
</tr>
</tbody>
</table>

\(^*\)\( n \) is the refractive index, \( k \) is the absorption coefficient, and \( \beta \) is the distribution’s surface roughness. For each material, the initial guesses for \( n \) and \( k \) came from the literature.\(^2\)
Bidirectional reflectance distribution functions (BRDFs) are considered physically plausible if they're based on valid assumptions of surfaces' physical structures (see Figure B). The most prominent model is the Torrance-Sparrow model (see the “Related Work” sidebar).

Most physically plausible BRDFs are based on the concept of microfacets. That is, a surface patch consists of many optically flat microfacets with varying facet normal orientation. Some microfacets block the light transport for certain incident and exitant angles—for example, by shadowing or masking.

So, such a BRDF consists of

- a geometry term \( G \) that models the shadowing and masking,
- a Fresnel term \( F_r \) for the fraction of light reflected from an optically flat microfacet, and
- a distribution term \( D \) to model the distribution of microfacets with a certain normal orientation distribution.

This last term is the most important because it defines the specular highlight’s brightness, size, and shape (see Figure B2).

We fitted the Fresnel terms with the parameters \( n \) (the refractive index) and \( k \) (the absorption coefficient) and the following distribution terms \( D \) for \( \beta \), the root-mean-square slope of the surface microfacets (the surface roughness). Note that \( \vec{n} \) and \( \vec{h} \) refer to the surface normal and microfacet normal—that is, the halfway-vector incident \( \vec{I} \) and exitant \( \vec{V} \) ray (see Figure B2).

For the Blinn-Phong distribution,

\[
D = \cos \left( \frac{\log(2)}{\beta} \right) \cos \left( \frac{\beta}{2} \right).
\]

For the Gaussian distribution,

\[
D = \exp \left( - \frac{\log(2)}{\beta} \cos \left( \frac{\beta}{2} \right) \right).
\]

That is, the facets are Gaussian distributed. \( D \) specifies the probability of a microfacet being oriented at an angle \( \left( \frac{\beta}{2} \right) \) from the average surface normal.

For the Beckmann distribution,

\[
D = \frac{1}{\beta^2 \cos \left( \frac{\beta}{2} \right)} \exp \left( - \left( \tan \left( \frac{\beta}{2} \right) \right)^2 \right).
\]

For the Trowbridge-and-Reitz distribution,

\[
D = \frac{c^2}{\left( \frac{\beta}{2} \right)^2 \left( c^2 - 1 \right) + 1}.
\]

This distribution models microfacets as ellipsoids with eccentricity \( c \), where

\[
c = \frac{\cos(\beta)^2 - 1}{\sqrt{\cos(\beta)^2 - 2}}
\]

describes the ratio of the lengths of the two main axes.

Table 1 in the main article shows the fitting results for copper, steel, aluminum, gold, and silver samples.

References
stated in the literature. Afterward, we tone-mapped the renderings with Durand’s algorithm\textsuperscript{5} for display in the article. Figure 8c plots the absolute per-pixel intensity difference on a logarithmic scale.

The gold images show the fewest differences. All the differences are subtle; that is, the surfaces computed with the ellipsometric measurement data from real-world surfaces fit visually with the predicted surfaces.

\textbf{Here, we verified the reflectance behavior for physically plausible models. However, we restricted ourselves to measuring a set of metallic surfaces. In the future, we will evaluate the reflectance behavior of dielectric materials (specifically, smooth and etched glass). Furthermore, we seek to measure the reflectance behavior for angles slightly differing from the mirroring angle over a considerable range.}
Acknowledgments
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Figure 8. Renderings of (from left to right) copper, silver, gold, and aluminum. (a) The surfaces with \( n \) and \( k \) fitted to our ellipsometric measurement data. (b) The predicted surfaces. (c) The per-pixel difference on a log scale. These graphs reveal that there are only subtle differences between the renderings based on our measurements and those based on the predictions.