Applying Fuzzy Relation Equations to Threat Analysis

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Abstract

Targeting behavior of vehicles in the battlefield (Target Analysis) is one of the most critical tasks in Computer Generated Forces (CGF) systems. This is simply because of many complex and ambiguous factors that can effect the targeting behavior of such systems in the real world. There has been many approaches including using Fuzzy Set Theory for Target Analysis. Target detection, and threat analysis and selection are considered the main constituents of Target Analysis and selection. In this paper we introduce a model to illustrate how to apply a fuzzy relational equation algorithm to threat analysis in context of Computer Generated Forces systems such ModSAF (Modular Semi Automated Forces) Using Fuzzy Relational Equations the proposed algorithm generates the data from the historic information and its earlier runs. Therefore, each new outcome of the algorithm is more realistic and more accurate than the earlier one.

1. BACKGROUND

The threat level posed by various targets, in real life, depends on various factors some are situation dependent and some related to characteristics of the target being analyzed, such are its formation, its firing status, etc. A great deal of work has been done by the Institute for Simulation and Training (IST) to identify those factors. The following is a list of the factors identified in [1]:

1. Aggregate Threat Assessment
2. Near Counter Threat
3. Target's Effective Range
4. Target Firing Status
5. Aspect Angle
6. Relative Elevation of Target
7. Target Movement
8. Target Type
9. Sector of Fire

Given the uncertainty and inaccuracy that is inherent in measuring/estimating the above factors, we use Fuzzy Set Theory to represent them. The following subsections, give a brief description of each of those factors. The reader is referred to [1] for a more detailed discussion.

Aggregate Threat Assessment

A target may be part of a unit and that unit could be a threatening or non-threatening unit. Targets belonging to a threatening unit are more threatening than the ones in non-threatening units. Factors considered in this category are:

1. Formation
2. Distance
3. Heading
4. Aiming Status
5. Closing Speed
6. Percentage of stationary targets

To estimate the unit's level of threat the above six factors must be considered. A detailed

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description of each of these factors can be found in [1].

Near Counter Threat

When there are friendly vehicles that can destroy the target as well then the threat level of the target will decline. However it would never become zero [1].

Target's Effective Range

If the platform is within the range of fire of the target then the threat level will increase. To model this the weapon's effective range and maximum range are used to create a sets of ranges analogous to following four threat levels: minimal, marginal, lethal, and deadly.

Target Firing Status

The behavior of a target affects its threat level. There are four sub-factors related to this factor which are:

1. Target has fired at the platform
2. Target is aiming at the platform
3. Target has simply fired
4. Target is scanning

For detailed description of these sub-factors refer to [1].

Aspect Angle

This factor indicates the direction an aircraft is aiming and is considered for aircraft only. The threat level increases if the aircraft is lined up to perform an attack.

Relative Elevation of Target

This factor is also important because depending on the combat scenario being in high or low elevation may increase the threat level. A set of fuzzy values used for this factor may be: "below", "level", "above", and "definitely above"[1].

Target Movement:

This is a dynamic factor that attempts to predict whether the target is becoming more threatening by moving toward the platform.

Target Type:

The type of target can definitely affect the threat level. However, for this factor the types need to be defined ahead time. If the target type can not be recognized then there must be other means to predict the type or to be on the safe side just set the default to the most threatening level.

Sector of Fire

The portion of the battlefield assigned to vehicle for scanning is called the Sector of Fire. If the platform is in a sector of fire of a target then the threat level for that target will increase

2. COMPUTING THE THREAT LEVEL

In this section, we present an algorithm for estimating the threat level Th of a target. Given a training set of fuzzy values of the factors and their corresponding threat levels, the algorithm establishes a fuzzy relation between the threat levels and the factors values. The generated realtion can then be used to estimate future threat levels from given factor values.

Defining the Association

Let **X** and **Y** be two finite sets and R a fuzzy relation from **X** into **Y**. Note that R is a fuzzy subset of the Cartesian product **X**×**Y** and can be represented by a matrix with coefficients in the interval [0,1], where the general entry R[x,y] is defined by the membership of (x,y) in R, for all $x \in \mathbf{X}$ and $y \in \mathbf{Y}$.

Let A and Th be two fuzzy membership functions in **X** and **Y**, respectively. We define the ♦ operator, for all $y \in \mathbf{Y}$, as:

$$[A \blacklozenge R](y) = \sup_{x \in X} [A(x) \wedge R(x,y)]$$

In our case, **Y** denotes a finite set of threat values. For example, $\mathbf{Y} = \{0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1\}$. And **X** denotes the set of attributes that effect Th. For example, we can define **X** to consist of the following attributes:

- Aggregate Threat Assessment (ATA)
- Near Counter Threat (NCT)
- Target's Effective Range (TER)
- Target Firing Status (TFS)

Note that A♦R is a fuzzy subset of **Y**. Moreover, based on our definition of the ♦operator, [A♦R](y) represents the strongest support to the belief that Th equals y. This stems from the sup-min composition and can be intuitively depicted in Figure 1. Note in the figure that the strength of each link between y and A is equal to the strength of its weakest segment (min. operation). The strength of the association between y and A is equal to the strength of its strongest link (sup operation). Therefore, we conclude that, for all $y \in \mathbf{Y}$, Th(y) = [A♦R](y); or more concisely:

$$Th = A \blacklozenge R \tag{1}$$

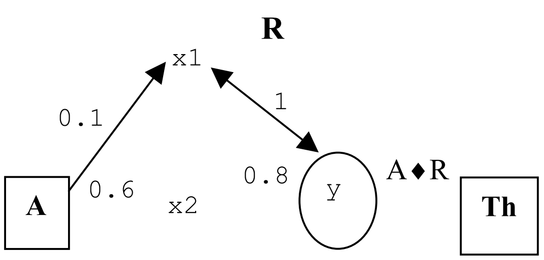
Equation (1) allows us to compute the threat value Th for a target given a fuzzy relation R and a set of attributes A for that target. Conversely, given the threat value of some target and its set of attributes A, one can solve equation (1) for R. We note first that equation (1) may have more than one solution; i.e. more than one relation may satisfy the equation for a given pair of A and Th. However, the maximal fuzzy relation that satisfies equation (1) is unique and is given by [2]:

$$\hat{R} = A \varphi Th \tag{2}$$

where φ is the fuzzy implication operator defined on [0,1]x[0,1] as:

$$a \ \varphi \ b = \sup\{c \mid 0 \le c \le 1, a \wedge c \le b\}$$

Equation (2) provides the strongest association between A and Th that satisfies equation (1). Note also than relation $\hat{R}$ can be interpreted as a set of fuzzy inference rules between A and Th.



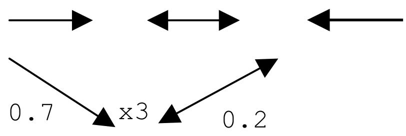


Figure 1: Graphical interpretation of the ♦ operator

Estimating the Maximal Fuzzy Relation

The previous section established an equation (equation (1)) for computing the threat level Th for a a target with a set of attributes A, given a fuzzy relation R between A and Th. Equation(2) gives the maximal fuzzy relation between a pair of A and Th that satisfies equation (1). In order to find a fuzzy relation R that can be used to compute/estimate the threat value for any target given his/her set of attributes, we need to establish a correspondence between certain sets of attributes and certain values of Th. The estimation problem can be formulated as follows:

Given a set of pairs of fuzzy sets $(A_1,Th_1),(A_2,Th_2),\ldots,(A_N,Th_N)$, estimate R such that the system of equations:

$$A_k \blacklozenge R = Th_k \tag{3}$$

is satisfied for all k=1,2,…,N

The given set of pairs can be viewed as a training set based upon which a relation R is estimated. Assuming the training set is well designed, the estimated relation can be used to compute the threat value for any target, given the target's set of attributes A.

As discussed in the previous section, each individual equation in the system of equations (3) may have more than one solution with the maximal solution given by equation (2). Let $\Re_k$ be the set of relations that satisfy the k^{th} equation of the system:

$$\Re_k = \{R \mid A_k \blacklozenge R = Th_k\}$$

Note that if $\bigcap_{k=1}^{N} \Re_k = \phi$, then the system of equations has no solution. However, if

$$\bigcap_{k=1}^{N} \Re_k \ne \phi \tag{4}$$

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it can be shown [2] that the maximal relation that satisfies the system of equations can be computed by intersecting all the fuzzy maximal relations that satisfy the individual equations; i.e.:

$$\hat{R} = \bigcap_{k=1}^{N} \hat{R}_k$$

where $\hat{R}_k = A_k \varphi Th_k$.

From a practical point of view, there is no simple way to verify condition (4). We therefore take the following three-step approach to estimating relation R:

1) Solve each equation of system (3) individually, using equation (1) to find the maximal relation $\hat{R}_k = A_k \varphi Th_k$

2) Compute $\hat{R} = \bigcap_{k=1}^{N} \hat{R}_k$

3) Verify that $\hat{R}$ as computed in step 2 satisfies each equation in 2 system (3). If it does not satisfy each equation, it is not a solution; otherwise, $\hat{R}$ is the maximal relation that can be established between the threat values of a target and its attributes.

Example

To illustrate the above algorithm, let's assume **X** and **Y** are defined as follows:

Y = {0, 0.2, 0.4, 0.6, 0.8, 1} and
X = {ATA, NCT, TER, TFS}

Where, as stated in section 3.1, the attributes are defined as follows:

- Aggregate Threat Assessment (ATA)
- Near Counter Threat (NCT)
- Target's Effective Range (TER)
- Target Firing Status (TFS)

We will use a vector notation for the fuzzy sets A and Th, with the supports implicitly in the order specified in the sets **X** and **Y** above. For example, A = {0.9/ATA, 0.1/NCT, 0.8/TER, 0.4/TFS} is written as A = [0.9, 0.1, 0.8, 0.4]. For the sake of brevity, we assume a small training set consisting of only the following two pairs of fuzzy sets:

- A_1 = [0.9, 0.1, 0.9, 0.2]
- Th_1 = [0.9, 0.7, 0.3, 0.2, 0.1, 0.1]
- A_2 = [0.1, 0.9, 0.1, 0.9]
- Th_2 = [0.1, 0.1, 0.4, 0.5, 0.9, 0.9]

Using equation (1), we get:

$$\hat{R}_1 = A_1 \varphi Th_1$$

$$\hat{R}_1 = [0.9, 0.1, 0.9, 0.2]\varphi \begin{bmatrix} 0.9 \\ 0.7 \\ 0.3 \\ 0.2 \\ 0.1 \\ 0.1 \end{bmatrix}$$

$$\hat{R}_1 = \begin{bmatrix} .9\varphi.9 & .9\varphi.7 & .9\varphi.3 & .9\varphi.2 & .9\varphi.1 & .9\varphi.1 \\ .1\varphi.9 & .1\varphi.7 & .1\varphi.3 & .1\varphi.2 & .1\varphi.1 & .1\varphi.1 \\ .9\varphi.9 & .9\varphi.7 & .9\varphi.3 & .9\varphi.2 & .9\varphi.1 & .9\varphi.1 \\ .2\varphi.9 & .2\varphi.7 & .2\varphi.3 & .2\varphi.2 & .2\varphi.1 & .2\varphi.1 \end{bmatrix}$$

$$\hat{R}_1 = \begin{bmatrix} 1 & .7 & .3 & .2 & .1 & .1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & .7 & .3 & .2 & .1 & .1 \\ 1 & 1 & 1 & 1 & .1 & .1 \end{bmatrix}$$

Using the same procedure we find:

$$\hat{R}_2 = A_2 \varphi Th_2 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ .1 & .1 & .4 & .5 & .9 & .9 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ .1 & .1 & .4 & .5 & .9 & .9 \end{bmatrix}.$$

Intersecting the two relations we get:

$$\hat{R} = \hat{R}_1 \cap \hat{R}_2 = \begin{bmatrix} 1 & .7 & .3 & .2 & .1 & .1 \\ .1 & .1 & .4 & .5 & .9 & .9 \\ 1 & .7 & .3 & .2 & .1 & .1 \\ .1 & .1 & .4 & .5 & .1 & .1 \end{bmatrix}$$

Finally, we verify whether $\hat{R}$ is actually a solution; i.e. whether the following equalities are satisfied:

$$A_1 \blacklozenge \hat{R} = Th_1 \text{ and } A_2 \blacklozenge \hat{R} = Th_2$$

For example,

$$A_1 \blacklozenge \hat{R} = \text{Sup}\left([.9, .1, .9, .2] \wedge \begin{bmatrix} 1 & .7 & .3 & .2 & .1 & .1 \\ .1 & .1 & .4 & .5 & .9 & .9 \\ 1 & .7 & .3 & .2 & .1 & .1 \\ .1 & .1 & .4 & .5 & .1 & .1 \end{bmatrix} \right)$$

Therefore, $A_1 \blacklozenge \hat{R}$ is equal to:

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$$= \begin{bmatrix} Sup[(.9 \wedge 1), (.1 \wedge .1), (.9 \wedge 1), (.2 \wedge .1)] \\ Sup[(.9 \wedge .7), (.1 \wedge .1), (.9 \wedge .7), (.2 \wedge .1)] \\ Sup[(.9 \wedge .3), (.1 \wedge .4), (.9 \wedge .3), (.2 \wedge .4)] \\ Sup[(.9 \wedge .2), (.1 \wedge .5), (.9 \wedge .2), (.2 \wedge .5)] \\ Sup[(.9 \wedge .1), (.1 \wedge .9), (.9 \wedge .1), (.2 \wedge .1)] \\ Sup[(.9 \wedge .1), (.1 \wedge .9), (.9 \wedge .1), (.2 \wedge .1)] \end{bmatrix}$$

$$= \begin{bmatrix} .9 \\ .7 \\ .3 \\ .2 \\ .1 \\ .1 \end{bmatrix} = Th_1$$

Having found a maximal relation R that satisfies the training set, we now can compute the threat level for any target, given its factor values.

Approximating the Maximal Fuzzy Relation

Based on our experimentation with the algorithm presented above, we noticed that in numerous cases the maximal relation $\hat{R}$ obtained in step 2 of the algorithm does not satisfy the system of equations but produces values for Th that are very close to the original values in the training set. In some cases, an approximate solution may be acceptable provided that the values of Th generated by the obtained relation are within an acceptable margin from the original values provided by the training set. Let ε denote the (acceptable) margin of error for the membership values defined by Th. An approximate solution to the system of equations for a given training set can be obtained by extending step 3 of the original algorithm as follows:

1) Solve each equation of system (3) individually, using equation (1) to find the maximal relation $\hat{R}_k = A_k \varphi Th_k$

2) Compute $\hat{R} = \bigcap_{k=1}^{N} \hat{R}_k$

3) If $(distance(Th_k, [A_k \blacklozenge \hat{R}]) \le \varepsilon)$ for all k = 1,2,…N, then accept $\hat{R}$ as a solution.

3. CONCLUSION

In this paper we presented a model to illustrate how to apply a fuzzy relational equation algorithm to perform threat analysis in the context of Computer Generated Forces systems such ModSAF (Modular Semi Automated Forces) Using Fuzzy Relational Equations the proposed algorithm generates the data from the historic information and its earlier runs. Therefore, each new outcome of the algorithm is more realistic and more accurate than the earlier one. The algorithm establishes a relation between a fuzzy set and its dependent factors. It can be used as a training algorithm to compute fuzzy sets based on a training set of the dependent attributes.

4. REFERENCES

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